MATHEMATICS DEPARTMENT

"A computer is the mathematicians best friend"

μ - Games
Mathematics Utrecht

Oktober 2021

Rules:

The idea of this event is to gap the bridge between mathematics and programming. When working on these exercises, we hope the participant will get a better understanding of the underlying mathematical concepts used. You will not be required to do a lot of difficult programming. With array manipulation and basic functionality you should be able to solve all the exercises.

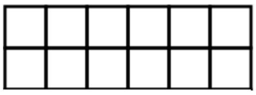
When working on these exercises, you must conform to the following rules.

- You are allowed to work in groups of maximum 4 persons.
- You will have three hours time.
- You can use no software packages outside from those specified.
- You can not use the internet except for handing in your solutions in DomJudge.

After the three hours, the solution to the exercises will be discussed and the solution has to be posted on the website *http://clover.science.uu.nl/dj*.

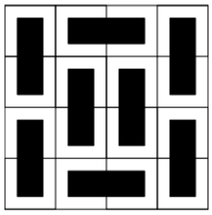
Problem 1
Tiling a band

We have the following problem. We have a $2 \times n$ rectangular grid, example as follows:

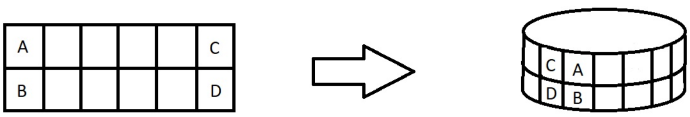


Suppose we have two types of tiles: a 2×1 or 1×2 rectangular blocks.

We say a "grid is cleared" if the blocks fill up the grid completely. Example as follows:



Now consider a "band" formed by joining the two breadths of the rectangular grid together. Example as follows:



We wish to know how many ways are there to clear such a grid-band formed from an arbitrary $2 \times n$ rectangular grid.

Input

- A number n, representing the length of a $2 \times n$ grid.

Output

- An integer, representing the number of tilings of a $2 \times n$ band.

Example

Input	Output
4	9

Problem 2
Particular Matrices

In this exercise you have to determine the determinant of a particular kind of matrix. As input you will get a matrix A of size n and a matrix T with the property

$$T_{ij} = 0 \text{ if } |i - j| > 1.$$

You will need to find the square root of the determinant of the matrix M given by

$$M = \begin{pmatrix} A & -T \\ T & \mathbf{0} \end{pmatrix}$$

Where 0 denotes the vector of size n with all entries equal to 0.

Input

- One line with the integer N, which represents the size of matrix A and T,
- N lines, each line containing a row of matrix A,
- N lines, each line containing a row of matrix T.

Output

- One line containing a number given by the square root of the determinant of the matrix M defined above.

Example

Input	Output
3 1 4 5 6 2 3 1 1 4 1 2 0 3 2 1 0 1 1	5

Problem 3
Nice Integral

In this exercise we will look at the moments of a particular random variable. Let X be an exponentially distributed random variable with rate parameter λ. This means that the probability density function f_X is given by

$$f_X(x) = \lambda e^{-\lambda x}, \text{ where } x \in [0, \infty).$$

Determine the n^{th}-moment $\mathbb{E}[(X-1)^n]$ for different values of n and λ.

Input

- A positive integer n,
- A positive number λ.

Output

- One line containing a number given by moment n of the random variable $X - 1$, where $X \sim Exp(\lambda)$. Round the number to the lowest integer.

Input	Output
10 2	479
15 4	22

Problem 4
Integer Factorization

Made by: Dr. S. (Stefano) Marseglia

Let N be a number, whose prime factorization consists of two big prime numbers p and q. We know that $p-1$ is a so-called smooth number, so $p-1$ is the product of "small" primes. $q-1$ is not smooth.
Compute the prime factorization of N.

Hint: If l is a prime number and a a positive integer coprime to l, then

$$a^{k(l-1)} \equiv 1 \bmod l,$$

for every integer k. Also, a good way to compute an integer which is divisible by all "small" primes (up to a certain bound) is to take $M!$ for M "big enough".

You can use the following algorithm to compute the GCD:

```
function gcd(a, b)
    while b ≠ 0:
        t = b
        b = a mod b
        a = t
    return a
```

Input

- One line with an integer we want to factorize.

Output

- One line with the biggest factor of the factorization.
- One line with the smallest factor of the factorization.

Example 1

Input	Output
323	19 17

Example 2[1]

Input	Output
41724500642360818244051685041472716218814916374189421825766872421309474411567426 7	

[1] *Note that in the table above, line 2 and 3 of the left column should be on one line. Since the numbers are big, this is the most efficient way to display this.*